

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics – 3

Subject Code : 4TE03EMT2

Branch: B. Tech (All)

Semester : 3

Date : 25/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
 - (2) Instructions written on main answer book are strictly to be obeyed.
 - (3) Draw neat diagrams and figures (if necessary) at right places.
 - (4) Assume suitable data if needed.
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Q-1

Attempt the following questions:

(14)

- a) The period of $\sin pt$ is
(A) 2π (B) $\frac{2\pi}{p}$ (C) $\frac{\pi}{p}$ (D) none of these
- b) In the Fourier series expansion of $f(x) = x^3$ in $(-1, 1)$
(A) only sine terms are present (B) both sine and cosine terms are present
(C) only cosine terms are present (D) constant term is present
- c) If $f(x) = x \cos x$ in $(-\pi, \pi)$ then the value of a_n equal to
(A) 0 (B) $\frac{\pi}{2}$ (C) $-\pi$ (D) 2π
- d) Laplace transform of $t^{\frac{1}{2}}$ is
(A) $\frac{\pi}{\sqrt{s}}$ (B) $\sqrt{\frac{\pi}{s}}$ (C) $\frac{\sqrt{\pi}}{s}$ (D) none of these
- e) $L^{-1} \tan^{-1}\left(\frac{1}{s}\right)$ is
(A) $\frac{\cos t}{t}$ (B) $\frac{\sin t}{t}$ (C) $\frac{\cot t}{t}$ (D) $\frac{\tan t}{t}$
- f) The Laplace transform of $tu(t-2)$ is
(A) $\left(\frac{1}{s^2} + \frac{2}{s}\right)e^{-2s}$ (B) $\frac{1}{s^2}e^{-2s}$ (C) $\left(\frac{1}{s^2} - \frac{2}{s}\right)e^{-2s}$ (D) $\frac{1}{s^2}e^{2s}$
- g) The C.F. of the differential equation $(D^3 + 2D^2 + D) = x^2$ is
(A) $y = c_1 + (c_2x + c_3)e^{2x}$ (B) $y = c_1 + (c_2 + c_3x)e^{-x}$ (C) $y = c_1 + (c_2x + c_3)e^x$
(D) none of these
- h) The P. I. of $(D^2 + 1)y = \cosh 3x$ is



- (A) $\frac{1}{10} \sinh 3x$ (B) $\frac{1}{5} \cosh 3x$ (C) $\frac{1}{10} \cosh 3x$ (D) none of these
- i) $\frac{1}{D-a} X$, (where $X = k$ is constant) equal to
 (A) $-\frac{k}{a}$ (B) $\frac{k}{a}$ (C) ka (D) $-ka$
- j) Eliminating the arbitrary constants, a and b from $z = (x+a)(y+b)$, the partial differential equation formed is
 (A) $z = \frac{p}{q}$ (B) $z = p+q$ (C) $z = pq$ (D) $xq = yp$
- k) The general solution of the equation $p \tan x + q \tan y = \tan z$ is
 (A) $F\left(\frac{\cos x}{\cos z}, \frac{\cos y}{\cos z}\right) = 0$ (B) $F(\sin x \sin y, \sin x + \sin y) = 0$
 (C) $F\left(\frac{\sin y}{\sin x}, \frac{\sin z}{\sin x}\right) = 0$ (D) none of these
- l) Particular integral of $(D^2 - D^2)z = \cos(x+y)$ is
 (A) $\frac{x}{2} \cos(x+y)$ (B) $x \sin(x+y)$ (C) $x \cos(x+y)$ (D) $\frac{x}{2} \sin(x+y)$
- m) The order of convergence in Bisection method is
 (A) zero (B) linear (C) quadratic (D) None of these
- n) The criterion for convergence for solving $f(x) = 0$ by the Newton-Raphson method is
 (A) $\left\{f'(x)\right\}^2 > |f(x) \cdot f''(x)|$ (B) $\left\{f'(x)\right\}^2 < |f(x) \cdot f''(x)|$
 (C) $\left\{f'(x)\right\}^2 = |f(x) \cdot f''(x)|$ (D) none of these

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) Using Newton-Raphson method, find the root of $f(x) = \sin x + \cos x$ correct to three decimal places. (5)
- b) One real root of the equation $x^3 - 4x - 9 = 0$ lies between 2.625 and 2.75. Find the root using Bisection method. (5)
- c) Evaluate: $L(te^{-4t} \sin 3t)$ (4)
- Q-3 Attempt all questions (14)**
- a) Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$. (5)
- b) Find the Fourier series for (5)



$$f(x) = a(x-l), \quad -l < x < 0$$

$$= a(l+x), \quad 0 < x < l$$

and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

- c) Given that one root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Find the root correct to 3 significant digits using Secant method. (4)

Q-4

Attempt all questions (14)

- a) Using Laplace transform method solve: (5)

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, \quad x(0) = 0, \quad x'(0) = 1$$

- b) Using convolution theorem, evaluate $L^{-1} \left\{ \frac{s}{(s^2 + 4)^2} \right\}$. (5)

- c) Solve: $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$. (4)

Q-5

Attempt all questions (14)

- a) Evaluate: $L^{-1} \left[\frac{1}{s^3 - a^3} \right]$ (5)

- b) Solve: $(D^2 - 2D + 1)y = xe^x \sin x$ (5)

- c) Solve: $pz - qz = z^2 + (x + y)^2$ (4)

Q-6

Attempt all questions (14)

- a) Solve: $D^2(D^2 + 4)y = 48x^2$ (5)

- b) If $f(x) = \begin{cases} x & , 0 < x < \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x < \pi \end{cases}$ then show that (5)

$$f(x) = \frac{4}{\pi} \left(\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right).$$

- c) Solve: $L \left(\frac{e^{-at} - e^{-bt}}{t} \right)$ (4)

Q-7

Attempt all questions (14)

- a) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + a^2y = \sec ax$ (5)

- b) Solve: $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$ (5)

- c) Solve: $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 2y$ (4)

Q-8

Attempt all questions (14)

- a) The following table gives the variations of periodic current $i = f(t)$ amperes over a period T sec. (7)



t (sec) :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
i (A) :	1.98	1.30	1.05	1.30	-0.88	-0.5	1.98

Show, by harmonic analysis, that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

- b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, (7)
given $u(x, 0) = 6e^{-3x}$

